

Composition of Functions

21. Two functions are given:

$$f(x) = \frac{x}{\sin x} \quad \text{and} \quad g(x) = x^2 - 2x + \frac{1}{x^2} - \frac{2}{x}.$$

(i) Determine the maximal (natural) domain of definition for both functions.

(ii) Show that $g\left(\frac{1}{f(x)}\right) = (g \circ f)(x)$. Provide a detailed justification.

22. Given the functions $f(x) = \frac{1}{x^2+1}$ and $g(x) = \sqrt{x}$, compute the formulas and domains of the compositions $f \circ g$ and $g \circ f$.

23. Let f and g be real functions of a real variable, defined as follows:

$$f(x) = \begin{cases} -1, & x < 1, \\ 1, & x \geq 1, \end{cases} \quad g(x) = \begin{cases} x^2, & x < 0, \\ x^2 - x - 1, & x \geq 0. \end{cases}$$

Determine the composition $f \circ g$.

24. Let f and g be real functions of a real variable, defined as follows:

$$f(x) = \begin{cases} -x^2 + 1, & x \geq 0, \\ e^x, & x < 0, \end{cases} \quad g(x) = \begin{cases} 1, & x < 1, \\ -1, & x \geq 1. \end{cases}$$

Determine the compositions $g \circ f$ and $f \circ g$.

25. Let f and g be real functions of a real variable, defined as follows:

$$f(x) = \begin{cases} x, & x < 0, \\ 0, & x \geq 0, \end{cases} \quad g(x) = \begin{cases} 1, & |x| \geq \frac{\pi}{2}, \\ |\sin x|, & |x| < \frac{\pi}{2}. \end{cases}$$

Write the rule for the composition $g \circ f$.

Rational Numbers

26. Prove that the number $\sqrt{5}$ is irrational.

27. Prove that the number $\sqrt{7}$ is irrational.

All above math problems are taken from the following website:

<https://osebje.famnit.upr.si/~penjic/teaching.html>.

THE READER CAN FIND ALL SOLUTIONS TO THE GIVEN PROBLEMS ON THE SAME PAGE.